ALGEBRA

List 4.

Rank of a matrix. Systems of linear equations

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1. Find the ranks of the matrices:

$$\text{(a)} \begin{pmatrix} 3 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & -1 & 1 \end{pmatrix}, \quad \text{(b)} \begin{pmatrix} -1 & 2 \\ 1 & -2 \\ -4 & 8 \end{pmatrix}, \quad \text{(c)} \begin{pmatrix} 2 & 0 & 2 & 4 & 6 \\ 1 & 2 & 0 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 4 & 1 & 2 & 1 & -2 \end{pmatrix}.$$

2. Using Cramer's rule, find the specified unknown in the SLE

$$\text{(a)} \begin{cases} 3x + z = 1 \\ y + 3z - 2x = 2 \\ -x + y - z = -1 \end{cases}, \text{ find } x, \quad \text{(b)} \begin{cases} -x + y - z = 1 \\ 2x - y - z = 2 \\ x + y + z = -3 \end{cases}, \text{ find } z,$$

$$\text{(c)} \begin{cases} x + z + y + t = 1 \\ x - z + y - t = 2 \\ x - z - y + t = -1 \\ x - z - y - t = 2 \end{cases}, \text{ find } y, \quad \text{(d)} \begin{cases} x + y + z + t = 2 \\ x + 2y + 2z + 2t = 3 \\ x + 2y + 3z + 3t = 4 \\ x + 2y + 3z + 4t = 5 \end{cases}, \text{ find } z.$$

3. Solve the SLEs from the previous problem using the Gauss elimination method.

4. Solve the SLEs

$$\text{(a)} \begin{cases} x + 2y + z = 3 \\ 3x + 2y + z = 3 \\ x - 2y - 5z = 1 \end{cases}, \quad \text{(b)} \begin{cases} x + 2y + 3z - 4v = 0 \\ 2x - y + 3z - 2v = 2 \\ 3x + 4z + 2v = -1 \end{cases}, \quad \text{(c)} \begin{cases} x + y + 2z - v = -1 \\ 2x + y + 3z + v = 3 \\ 3x + y - z - 2v = -4 \end{cases}$$

5. (a) Show that the SLE

$$\begin{cases} x + 2y + z + 4v = 1 \\ 2x + y + 3v = 3 \\ -x + 2z + v = 1 \\ 2x + y + 3z + 6v = 6 \end{cases}$$

is inconsistent.
(b) Find all values of the parameters a, b, c, d such that the SLE

$$\begin{cases} x + 2y + z + 4v = a \\ 2x + y + 3v = b \\ -x + 2z + v = c \\ 2x + y + 3z + 6v = d \end{cases}$$

is consistent.
(c) For the SLE above, in case it is consistent, determine the dimension of the set of its solutions.

6. Decompose the rational functions into real partial fractions:

$$\text{(a)} \frac{x^3 + 2x^2 + 3x - 4}{(x^2 + x + 1)(x^2 + 2x + 5)}, \quad \text{(b)} \frac{x^5 + x^4 - 2x^2 + 1}{(x^2 + x + 1)(x^2 + x + 2)(x^2 + 2x + 5)},$$

$$\text{(c)} \frac{x^3 + 2x^2 + 3x - 4}{(x^2 + x + 1)^2(x^2 + 2x + 5)}.$$